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ESTIMATE OF THE MASS OF DARK MATTER IN INHOMOGENEOUS UNIVERSE

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Abstract

Attempts have been made to study the fundamental properties of dark matter and structure formation of the inhomogeneous cosmology. Detailed mechanism of the relation of dark matter candidates in the range of inflation of the universe and axion decay rate at inflation has been investigated and the mass of axion, one of the major candidates of dark matter, has been to estimate in the frameworks of general relativity and elementary particle. As the situation dicates, Mathematica software is utilized for detailed computations and visualization of the results.

Keywords: dark matter, inhomogeneous cosmology, axion

Introduction

The term "dark matter" was first introduced by Fritz Zwicky (1933) in his term paper when he studied the dynamics of galaxy clusters. Several powerful astrophysical and cosmological arguments which dislike "baryonic matter" as a main constituent constrained the physical properties of dark matter. The most popular argument is that of "particle dark matter" which is speculated by Cowski and McClelland (1973) that the new matter appears to hold galaxies and galaxy clusters gravitationally together rather than the baryonic matter.

In the last two decades, new and powerful techniques have emerged that the universe is filled with dark matter. A standard model of cosmology is emerging in which the universe is made of 4.9% of ordinary baryonic matter, nearly 26.8% of dark matter and 68.3% of dark energy according to NASA/WMAP Science Team. Based on the current research, scientists know only that dark matter is cold, slow moving, and interacts weakly with ordinary matter.

The quantity and composition of matter and energy in the universe is a fundamental and important issue in cosmology and also crucial for understanding the past as well as the future of the universe. The growth of small inhomogeneities in the matter and ultimately how large scale structure formed in the present age of the universe, the formation and evolution of individual galaxies, can be determined.

Structural Formation of the Inhomogeneous Universe

From a spatially flat Friedman-Robertson-Walker (FRW) model, it can be induced by the inhomogeneous dark fluids coupled with dark matter in the equation of state. Starting from a universe filled with two interacting ideal fluids, dark energy and dark matter with scale factor. The governing equations are

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$$\dot{\rho} + 3H(p+\rho) = -Q$$
$$\dot{\rho}_m + 3H(p_m + \rho_m) = Q \qquad (1)$$
$$\dot{H} = -\frac{k^2}{2}\left(p + \rho + p_m + \rho_m\right)$$

where, the hubble rate H and $k^2=8\pi G$, G is Newton's gravitational constant and p and ρ are the pressure and the energy density respectively and Q is the function that accounts for the energy exchange between dark energy and dark matter.

Friedman's equation for the Hubble rate is given by

$$H^2 = \frac{k^2}{3}\left(\rho + \rho_m\right)$$

For the case study of nonrelativistic dark matter ($\overline{\omega} = 0$), and the consideration the dark matter as a dark, then one obtains $p_m = 0$.

Now the gravitational equation of motion for dark matter becomes

$$\dot{\rho}_m + \sqrt{3}k\rho_m\sqrt{\rho + \rho_m} = Q \qquad (2)$$

The relation between Q and H is

$$Q = \delta H^2$$

Where δ is positive constant and it can be obtained Q for the existence of scaling solutions as

$$Q = \frac{\delta\gamma^2}{3}\rho_m \text{ where } \gamma = \frac{k}{\sqrt{3}}\sqrt{1+\frac{1}{r}} \text{ and } r = \frac{\rho_m}{\rho}.$$

In this situation the energy density of dark matter is given by

$$\rho_m = \left(\frac{C\delta\gamma}{3C + e^{\frac{-\eta^2}{2}}}\right), \qquad (3)$$

where C is an integration constant, $\eta = \delta\gamma^2$.

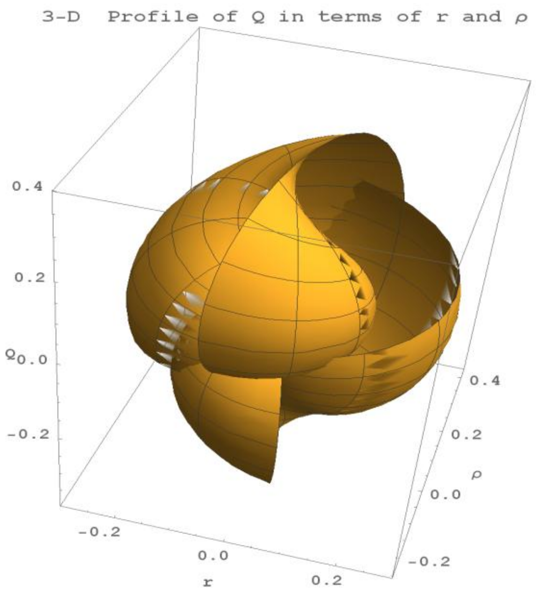

Figure 2 3-D profile of Q in terms of r and ρ

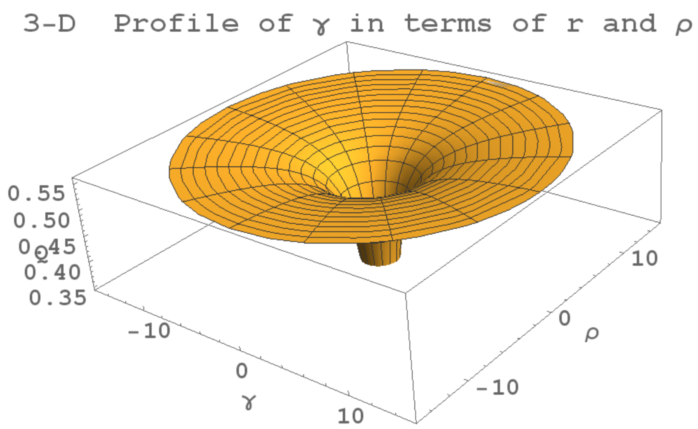

Figure 2 3-D profile of γ in terms of r and ρ

Overview of Dark Matter Candidate

There are many possible candidates of dark matter assuming in modern cosmology ranging the mass from axion with mass 10^{-5} eV to black holes of mass which are 10^4 times that of solar mass. The first class is to classify the baryonic and non-baryonic. The main baryonic matters are massive compact halo objects (MACHOs) including brown dwarfs, jupiters, stellar black-hole remnants, white dwarfs, and neutron stars, etc. The remaining of the dark matter candidates is non-baryonic which can be in further classification with hot and cold dark matter. The hot dark matter is moving at relativistic speeds when galaxies could just start to form. The major assumption of hot dark matter candidates is neutrino. However, hot dark matter does poorly in reproducing the N body simulations of structure formation in a universe. There have been other suggestions that most of part is cold dart matter.

The non-baryonic cold dark matter candidates can be assumed basically elementary particles although they have not been discovered. The axion and weakly-interacting massive particles (WIMPs) are the leading class in these candidates. There are other pseudo-Nambu-Goldstone bosons which are similar to the axion which have been also proposed as dark matter candidates.

Axion dynamics

In the standard model, the sum of the QCD topological angle and the common quark mass phase $\theta = \theta_0 + \arg\det M$ is experimentally bounded to lie below from the non-observation of the neutron electric dipole moment. Among the known solutions, the QCD axion is probably the most simple and the standard model becomes an argument with an extra pseudo-goldstone boson which only derivative coupling to QCD topological control and repressed by the scale f_A. This coupling permit the effects of θ to be redefined away via a shift of the axion field, whose vacuum expectation value (VEV) is then guaranteed to vanish. It can also produce the mass of the axion. Another model dependent derivative coupling may be present but they do not affect the solution of the strong CP problem, but the mass and couplings of the QCD axion are thus controlled by a single scale f_A.

At energies lower than the PQ and electroweak EW breaking scales, the axion dependent part of the Lagrangian and the weak couplings can be written as

$$\mathcal{L}_a = \frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{a}{f_A}\frac{\alpha_s}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4} a g^0_{a\gamma\gamma} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu a}{2 f_A} j^\mu_{a,0} \quad (4)$$

In the above equation the second term defines f_A with the dual gluon field strength $\tilde{G}_{\mu\nu} = \frac{1}{2}\in_{\mu\nu\rho\sigma}$ colour indices which is implicit and the coupling to the photon field strength $F_{\mu\nu}$ is

$$g^0_{a\gamma\gamma} = \frac{\alpha_{EM}}{2\pi f_A}\frac{E}{N} \quad (5)$$

where $\frac{E}{N}$ is the ratio of electromagnetic EM and the colour incongruity.

The last term of equation(4)

$$j^\mu_{a,0} = c^0_q \bar{q}\,\gamma^\mu \gamma_5 q \quad (6)$$

is a model dependent axial current made of SM matter fields . The axionic pseudo shift symmetry $a \to a + \delta$ can remove QCD θ angle.

In particular performing a charge of field variables on top and down quarks

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \to e^{i\gamma_5 \frac{a}{2 f_a} Q_a} \begin{pmatrix} u \\ d \end{pmatrix}, \text{and Tr } Q_a = 1 \quad (7)$$

Equation (4) becomes

$$\mathcal{L}_a = \frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{1}{4} a g^0_{a\gamma\gamma} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu a}{2 f_a} j^\mu_{a,0} - \bar{a}_L M_a a_R + h.c \quad (8)$$

where ,

$$g_{a\gamma\gamma} = \frac{\alpha_{EM}}{2\pi\, f_a}\left[\frac{E}{N} - 6\, Tr\ (Q_a\, Q^2)\right] \tag{9}$$

$$j_a^{\mu} = j_{a,0}^{\mu} - \bar{q}\ \gamma^{\mu}\ \gamma_5\ Q_a\, q \tag{10}$$

$$M_a = e^{i\frac{a}{2f_a}Q_a} M_q\, e^{i\frac{a}{2f_a}Q_a} \tag{11}$$

$$M_q = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \text{ and } Q = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \tag{12}$$

This axion coupling to the axial current only renormalizes multiplicatively and the axion appears through the quark mass terms in this.

The axion can be preserved as an external source by the leading $\frac{1}{f_A}$, and the virtual axions being affected by the tiny coupling. The non-derivative couplings to QCD are programmed in the phase dependence of the quark mass matrix M_a which in the derivative couplings the axion be an external axial current. At the above equation, the choice of axion field allowed to move the non-derivative couplings entirely into the lightest two quarks. From these quarks, it can be integrated out all of the other quarks and directly work in the effective theory with M_a capturing with the whole axion dependence.

In the chiral expansion, all the non-derivative dependence on the axion is contained in the pion mass terms;

$$\mathcal{L}_{p2} \supset 2B_0 \frac{f_\pi^{\ 2}}{4}\left\langle UM_a^{\ +} + M_a U^{+}\right\rangle \qquad \text{c} \tag{13}$$

where, $U = e^{i\beta/f_\pi}$ and $\beta = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$

< > is the traces over flavor indices and B_0 is related to the chiral condensate and determine by the pion mass in terms of quark masses and the pion decay constant is normalized such that $f_\pi \approx 92MeV$.

In order to derive the leading order effective axion potential, it is required to only consider the neutral pion sector. Choosing Q_a proportional to the identity,

$$V(a,\pi^0) = -B_0\, f_\pi^2\left[m_u \cos\left(\frac{\pi^0}{f_\pi} - \frac{a}{2f_A}\right) + m_d \cos\left(\frac{\pi^0}{f_\pi} + \frac{a}{2f_A}\right)\right]$$

To be solved in all closed form at all $\left(\frac{a}{2f_A}\right)$, it can be considered the case of flavours as

$$\phi_u = \bar{\phi}_u = \arctan \frac{m_d \sin\left(\frac{a}{2f_A}\right)}{m_u + m_d \sin\left(\frac{a}{2f_A}\right)}$$

$$\phi_d = \bar{\phi}_d = \arctan \frac{m_u \sin\left(\frac{a}{2f_A}\right)}{m_d + m_u \sin\left(\frac{a}{2f_A}\right)}$$

and, $\tan\phi_a = \frac{m_u - m_d}{m_u + m_d} \tan\left(\frac{a}{2f_A}\right)$, where $\phi_a = \bar{\phi}_u + \bar{\phi}_d$

$$V(a,\pi^0) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_A}\right)} \cos\left(\frac{\pi^0}{f_\pi} - \phi_a\right) \quad (14)$$

On the vacuum π^0 acquires a vacuum expectation value (VEV) proportional to ϕ_a which can be minimized the potential, the last cosine in equation(11) is 1 on the vacuum and π^0 can be trivially integrated out leaving the axion effective potential.

$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_A}\right)} \quad (15)$$

At the minimum $\langle a \rangle$=0, and mounting to quadratic order, then we have the axion mass equation

$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_A^2} \quad (16)$$

Axion Inflation

At the end of the inflation, the axion is to be considered the initial value of $a_0=f_0\theta_0$, where the angle θ_0 it profits the value in the range of $[0,2\pi)$.When it is considered the PQ symmetry is broken sufficiently at the ahead of inflation, the initial value ends up being constant on super horizon scales, and it can be defined as $a_0=f_A$ and treat f_A as a free parameter. Predicting that the final baryon asymmetry will depend on a_0, the quantum fluctuations of the axion field during inflation be smaller than the observational upper limit.This implies a constraint on the Hubble rate during inflation,

$$H_{\inf} \le 6\times 10^{11} GeV\left(\frac{f_A}{10^{15} GeV}\right) \quad (17)$$

The evolution of the homogeneous axion field in effective potential V $_{\text{eff}}$ around the origin can be resulting as follow.

The progression of the axion field in the expanding universe is defined as

$$\ddot{a}(x)+3H\dot{a}(x)-\frac{\nabla^2}{R^2(t)}a(x)+\frac{dV(a)}{da}=0 \tag{18}$$

where, R(t) is the universal scale factor. The effective potential for the axion field is

$$V_{full}(a)=V_{QCD}(a)+V_{Grav}(a)$$

$$=m_a^2F_a^2\left(1-\cos\left(\frac{a}{F_a}\right)\right)+V_{Grav}(a) \tag{19}$$

$V_{Grav}(a)$ is unknown due because of not having the compressive theory, but it can be supposed that $V_{Grav}(a)$ is not be affected and is to be negligible compared with $V_{QCD}(a)$.

Equation (15) becomes

$$\ddot{a}(x)+3H\,\dot{a}(x)-\frac{\nabla^2}{R^2(t)}a(x)+m_a^2\,F_a^2=0 \tag{20}$$

assuming a is small compared to F_a.

Let us consider the time t_1 at the following condition

$m_a(T_1)=3H(t_1)$, the Hubble parameter at that time t_1 with the temperature T_1.Using the temperature dependence of $m_a(T_1)$ and for t< t_1, it can be unnoticed the mass term due to $H>>m_a$.

When the axion mass term can be considered, they begin to oscillate with a frequency $\approx m_a$, calling the condition as zero mode and marked as a_0

$$\ddot{a}_0+3H\dot{a}_0+m_a^2a_0=0 \tag{21}$$

From equation (17) and (18), it is taken as $\frac{a_0}{f_A}$ is 1 in the entire universe at the end of inflation to be reliability .When the inflation is over, the axion field is at rest until the Hubble parameter becomes $H_{osc}=m_a$.

In the early universe, after PQ transition, the axion field grows according to equation (21) $\ddot{a}_0+3H\dot{a}_0+\frac{\chi}{f_A^2}a_0=0$.After the end of inflation, the temperature rises to its maxima value after which it decreases because the energy density is dominant by the inflaton. With the axion decay rate

$$\Gamma_a\approx\frac{\left(\frac{g_2^2}{4\pi}\right)^2}{64\pi^3}\frac{m_a^2}{f_A^2},$$

and then it is recruit that the secondary reheating temperature or axion decay temperature

$$T_{dec} \approx 1\times10^{4} GeV\left(\frac{m_a}{10^{9} GeV}\right)^{\frac{3}{2}}\left(\frac{10^{15} GeV}{f_A}\right) \quad (22)$$

At temperatures around a GeV, when $\frac{\sqrt{\chi(t)}}{f_A} \approx 3H(t)$, the axion field becomes proposed to the minimum value of the potential and indications to oscillate around CP conserving ground state. Thus, the condition has an equation of state as cold dark matter, $\overline{\omega} \approx 0$ whereas dark matter is restrained as dust in matter dominant universe. Taking the average values of axion field in that temperature it is initiate as in the vacuum realignment mechanism.

$$\Omega_A^V h^2 \approx 3.8\times10^{-3}\left(\frac{f_A}{10^{10} GeV}\right)^{1.165} \quad (23)$$

The value 1.165 arises from the temperature dependent of $\chi(t)$ determined from lattice QCD.(Borsanyli,S.,2016)

Axion will also be produced by topological defects and domain walls which are designed at the boundaries of the domains. According the states of this domains, most of the axions produced from strings during the radiation dominated era as (Ballesteros,G.,2016)

$$\Omega_A^S h^2 \approx 7.8\times10^{-3}\left(\frac{f_A}{10^{10} GeV}\right)^{1.165} \quad (24)$$

Importantly, strings are always attached by N_{DW} domains walls, subsequently the value of phase PQ field around the string core, and then the networks of strings and domain walls called string wall systems are formed. The string wall systems in short-lived (N_{DW}=1), their collapse donates an amount to dark matter as (Kawasaki, M., 2015)

$$\Omega_A^C h^2 \approx 3.9\times10^{-3}\left(\frac{f_A}{10^{10} GeV}\right)^{1.165} \quad (25)$$

Now, the total abundance of dark matter is

$$\Omega_A h^2 \approx \left(\Omega_A^V h^2 + \Omega_A^S h^2 + \Omega_A^C h^2\right) \approx 15.5\times10^{-3}\left(\frac{f_A}{10^{10} GeV}\right)^{1.165} \quad (26)$$

The best measurement of matter currently have in the present universe is (Plehn,T.,2017)

$$\Omega_m h^2 \approx 0.1198$$

Then it can be taken as by comparing equation (26),

$$f_A \approx 1.7275\times10^{9} GeV$$

Inserting this value into equation (16), the mass of axion can be estimated.

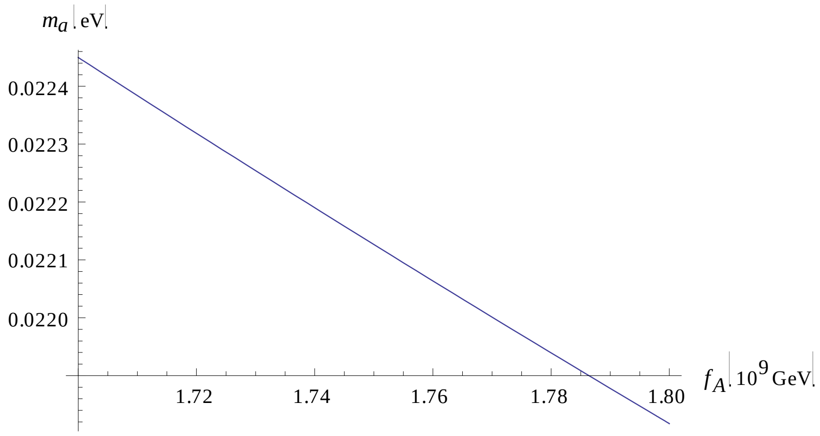

Figure 3 The estimate range of axion mass dependent on f_A in the range of (1.7~1.8) x 10^9GeV

Concluding Remarks

It has been attempted, in this paper, to clarify the basic ideas to understand the axion dark matter and its related ramifications. From the structure formation of the inhomogeneous cosmology, it is observed that the function between matter and energy shows the nature of spiral. The axion dynamics has been investigated using QCD variables with the dependent part of the Lagrangian and the coupling f_a, and the chiral expansion. Detailed mechanism of the relation of dark matter candidate in the range of inflation of the universe and axion decay rate at inflation has been investigated and the mass of axion has also been estimated under the classical motion during inflation and reheating of the universe accompanied with the imposing of scalar field. According to this current research, it is found that the mass of axion is dependent on the scale factor and have the estimate of mass of axion is within the range of < 22.6 meV. This result is in well agreement with the one given in the recent result, 0.56meV < m_a < 130(4.5)meV. (Ringwald A, 2018)

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